

# **Non-interactive Blind Signatures for Random Messages**

*Lucjan Hanzlik*

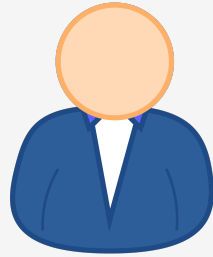
Eurocrypt 2023, Lyon





# Two-move Blind Signatures

**User/Recipient**



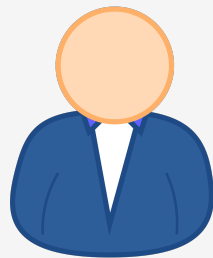
**Signer**





# Two-move Blind Signatures

**User/Recipient**



$(req, St) \leftarrow \text{Request}(m, pk)$

**Signer**





# Two-move Blind Signatures

User/Recipient



$(req, St) \leftarrow \text{Request}(m, pk)$

req



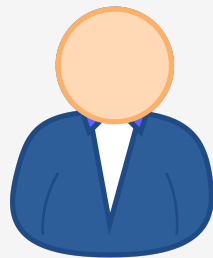
Signer





# Two-move Blind Signatures

**User/Recipient**



$(req, St) \leftarrow \text{Request}(m, pk)$

req



**Signer**

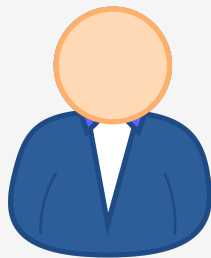


$pre \leftarrow \text{Issue}(req, sk)$



# Two-move Blind Signatures

User/Recipient



$(req, St) \leftarrow \text{Request}(m, pk)$

req

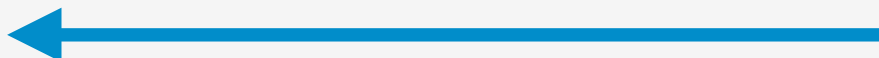


Signer



$pre \leftarrow \text{Issue}(req, sk)$

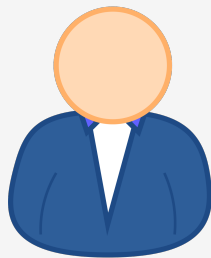
pre





# Two-move Blind Signatures

User/Recipient



$(req, St) \leftarrow \text{Request}(m, pk)$

req

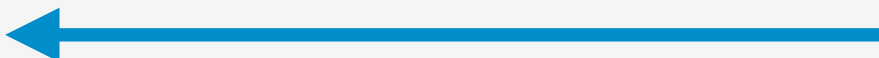


Signer



$pre \leftarrow \text{Issue}(req, sk)$

pre

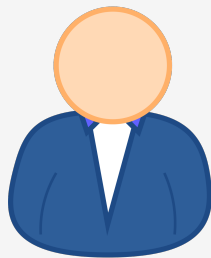


$sig \leftarrow \text{Obtain}(pre, St, pk)$



# Two-move Blind Signatures

User/Recipient



$(req, St) \leftarrow \text{Request}(m, pk)$

req

Signer

Unforgeability

$pre \leftarrow \text{Issue}(req, sk)$

pre

$sig \leftarrow \text{Obtain}(pre, St, pk)$



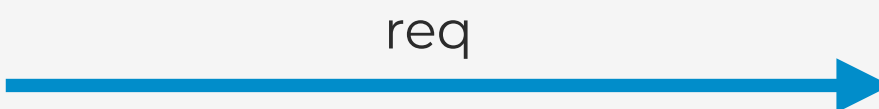


# Two-move Blind Signatures

User/Recipient



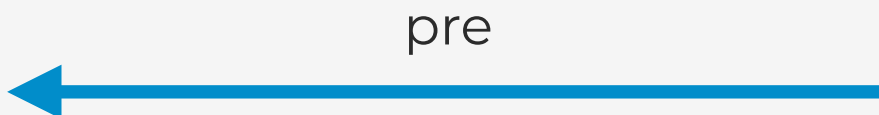
$(req, St) \leftarrow \text{Request}(m, pk)$



Signer



$pre \leftarrow \text{Issue}(req, sk)$



$sig \leftarrow \text{Obtain}(pre, St, pk)$



# Chaum's E-cash

**User**



**Bank**



**Merchant**



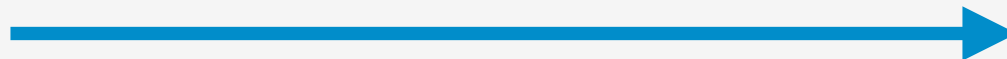


# Chaum's E-cash

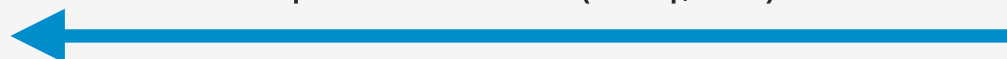
**User**



$(req, St) \leftarrow \text{Request}(m, pk)$



$pre \leftarrow \text{Issue}(req, sk)$



**Bank**



$sig \leftarrow \text{Obtain}(pre, St, pk)$

**Merchant**



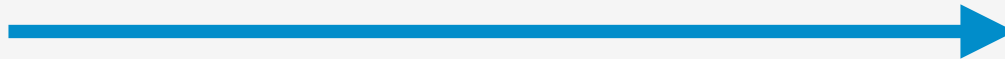


# Chaum's E-cash

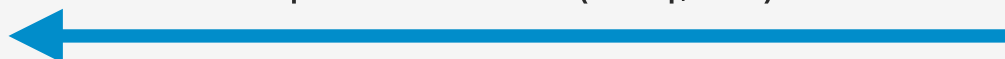
User



$(req, St) \leftarrow \text{Request}(m, pk)$



$pre \leftarrow \text{Issue}(req, sk)$



Bank



$sig \leftarrow \text{Obtain}(pre, St, pk)$



$(m, sig)$

Merchant





# Chaum's E-cash

**User**



$(req, St) \leftarrow \text{Request}(m, pk)$



$pre \leftarrow \text{Issue}(req, sk)$



**Bank**



$sig \leftarrow \text{Obtain}(pre, St, pk)$



$(m, sig)$

**Merchant**

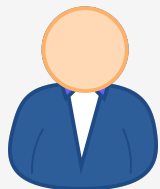


$\text{Verify}(m, sig, pk)$

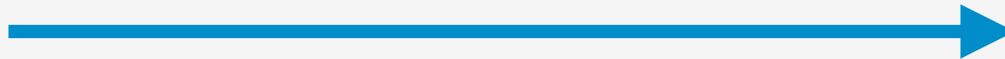


# Chaum's E-cash

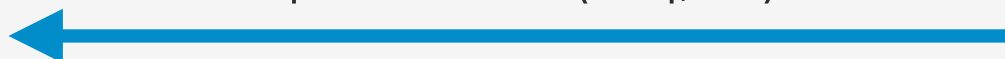
User



$(req, St) \leftarrow \text{Request}(m, pk)$



$pre \leftarrow \text{Issue}(req, sk)$



Bank



$sig \leftarrow \text{Obtain}(pre, St, pk)$

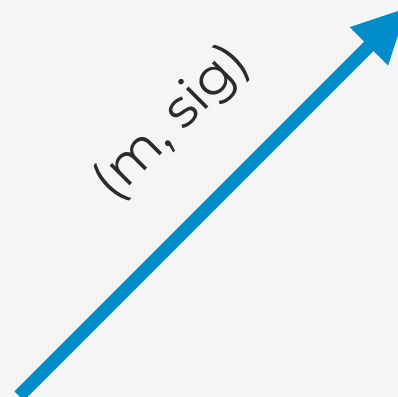


$(m, sig)$

Merchant



$\text{Verify}(m, sig, pk)$



$(m, sig)$



# Chaum's E-cash

User



$(req, St) \leftarrow \text{Request}(m, pk)$

$pre \leftarrow \text{Issue}(req, sk)$

$sig \leftarrow \text{Obtain}(pre, St, pk)$

$(m, sig)$

Merchant



$\text{Verify}(m, sig, pk)$

Bank



$\text{Verify}(m, sig, pk)$

$(m, sig)$



# Chaum's E-cash

User



$(req, St) \leftarrow \text{Request}(m, pk)$

$pre \leftarrow \text{Issue}(req, sk)$

$sig \leftarrow \text{Obtain}(pre, St, pk)$

$(m, sig)$

Merchant



$\text{Verify}(m, sig, pk)$

Bank



$\text{Verify}(m, sig, pk)$

Store m

$(m, sig)$





# Chaum's E-cash - other scenarios



# Chaum's E-cash - other scenarios



Privacy Pass



# Chaum's E-cash - other scenarios



Privacy Pass

## TumbleBit: An Untrusted Bitcoin-Compatible Anonymous Payment Hub

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†George Mason University foteini@gmu.edu

‡North Carolina State University ascafur@ncsu.edu



# Chaum's E-cash - other scenarios

In all scenarios messages  
are random strings.

Can we use this?

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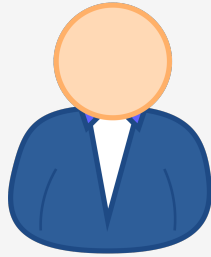
†George Mason University foteini@gmu.edu

‡North Carolina State University ascafur@ncsu.edu



# Two-move Blind Signatures for Random Messages

User/Recipient



$(req, St) \leftarrow \text{Request}(pk)$

req

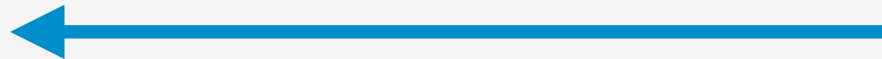


Signer



$pre \leftarrow \text{Issue}(req, sk)$

pre

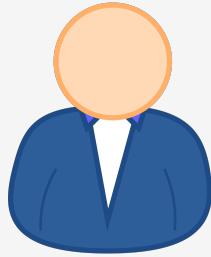


$(m, sig) \leftarrow \text{Obtain}(pre, St, pk)$



# Two-move Blind Signatures for Random Messages

User/Recipient



$(req, St) \leftarrow \text{Request}(m)$

Signer



$pre \leftarrow \text{Issue}(req, sk)$

**Not really interesting.  
Efficient two-move BS exist  
and provide more features.**

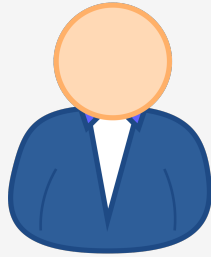
pre

$(m, sig) \leftarrow \text{Obtain}(pre, St, pk)$



# Two-move Blind Signatures for Random Messages

User/Recipient



$(req, St) \leftarrow \text{Request}(r)$

Signer



**Not really interesting.  
Efficient two-move BS exist  
and provide more features.**



**Do we need interaction if user does  
not pick the message?**

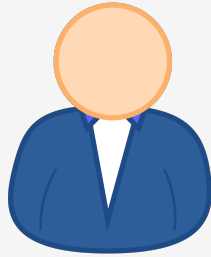
$(sk) \leftarrow \text{Issue}(req, sk)$

$(m, sig) \leftarrow \text{Obtain}(pre)$



# Strawman Solution #1

User/Recipient



Signer



$pre \leftarrow \text{Issue}(sk)$

$pre$



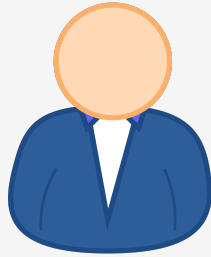
$(m, sig) \leftarrow \text{Obtain}(pk, pre)$





# Strawman Solution #1

User/Recipient

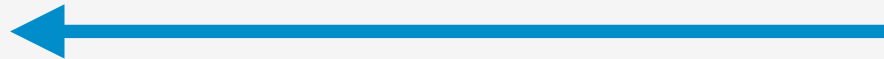


Signer



$pre \leftarrow \text{Issue}(sk)$

pre



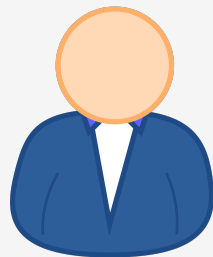
$(m, sig) \leftarrow \text{Obtain}(pk, pre)$

**User can unblind  
presignature  
many times!**



# Strawman Solution #2

User/Recipient (**skr**, pkr)



Signer



$pre \leftarrow \text{Issue}(sk, pkr)$

pre

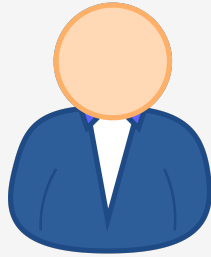


$(m, sig) \leftarrow \text{Obtain}(skr, pkr, pre)$



# Strawman Solution #2

User/Recipient (**skr**, pkr)



Signer



$pre \leftarrow \text{Issue}(sk, pkr)$

pre



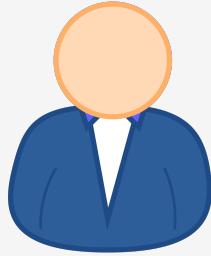
$(m, sig) \leftarrow \text{Obtain}(skr, pkr, pre)$

**Message m is now a  
function of skr**



# Strawman Solution #2

User/Recipient (**skr**, pkr)



Signer



$pre \leftarrow \text{Issue}(sk, pkr)$

pre



$(m, sig) \leftarrow \text{Obtain}(skr, pkr, pre)$

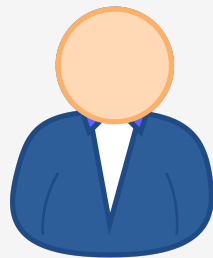
**Message m is now a  
function of skr**

**Only one presignature  
per pkr**



# Solution #3

User/Recipient (skr, pkr)



Signer



$pre \leftarrow \text{Issue}(sk, pkr, \text{nonce})$

$pre, \text{nonce}$



$(m, sig) \leftarrow \text{Obtain}(skr, pkr, pre)$



# Non-interactive Blind Signatures (NIBS)

**KeyGen(secpar)**

*outputs signer's key pair (sk,pk)*

**RKeyGen(secpar)**

*outputs recipient's key pair (skr,pkr)*

**Issue(sk,pkr,nonce)**

*outputs presignature (pre)*

**Obtain(skr,pk,pre,nonce)**

*outputs message-signature pair (m, sig)*

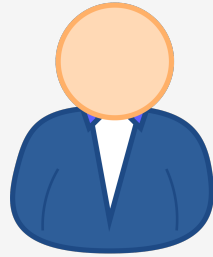
**Verify(pk, (m,sig) )**

*outputs validity of message-signature pair*



# How to use NIBS?

**User/Recipient**



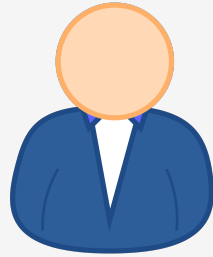
**Signer**





# How to use NIBS?

**User/Recipient**



$(\text{skr}, \text{pkr}) \leftarrow \text{RKeyGen}(\text{secpar})$

**Signer**

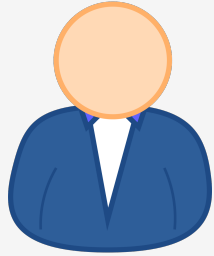






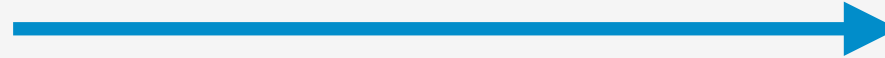
# How to use NIBS?

User/Recipient



$(\text{skr}, \text{pkr}) \leftarrow \text{RKeyGen}(\text{secpar})$

pkr



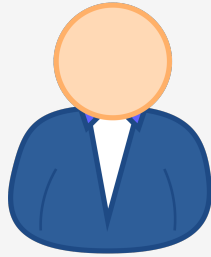
Signer





# How to use NIBS?

User/Recipient



$(\text{skr}, \text{pkr}) \leftarrow \text{RKeyGen}(\text{secpar})$

Signer



$\text{pkr}$



$\text{pre}, \text{nonce}$

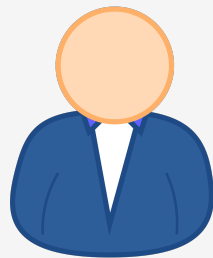


$\text{pre} \leftarrow \text{Issue}(\text{sk}, \text{pkr}, \text{nonce})$



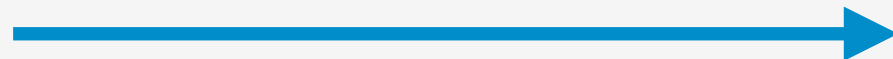
# How to use NIBS?

User/Recipient

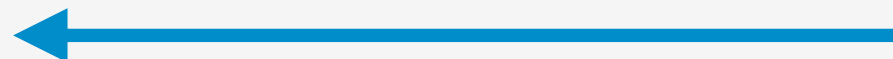


$(\text{skr}, \text{pkr}) \leftarrow \text{RKeyGen}(\text{secpar})$

$\text{pkr}$



$\text{pre}, \text{nonce}$



$\text{pre}_2, \text{nonce}_2$



Signer



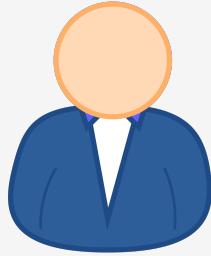
$\text{pre} \leftarrow \text{Issue}(\text{sk}, \text{pkr}, \text{nonce})$

$\text{pre}_2 \leftarrow \text{Issue}(\text{sk}, \text{pkr}, \text{nonce}_2)$



# How to use NIBS?

User/Recipient



$(skr, pkr) \leftarrow RKeyGen(secpar)$

Signer



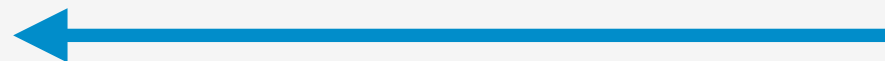
**Standard PKI keys  
used in one of the  
schemes.**

pre, nonce



$pre \leftarrow Issue(sk, pkr, nonce)$

pre<sub>2</sub>, nonce<sub>2</sub>



$pre_2 \leftarrow Issue(sk, pkr, nonce_2)$



# Applications

- All e-cash scenarios including Privacy Pass  
(Batch issuing with a single message)
- Lottery System  
(Final message unpredictable)
- Whistleblowing System  
(Using existing PKI to distribute tokens  
that can be later redeemed)
- Airdropping E-cash  
(E-cash systems can send  
free tokens to users)



# Unforgeability for NIBS

**Adversary**



**Challenger**



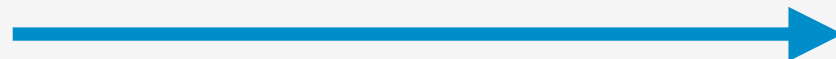


# Unforgeability for NIBS

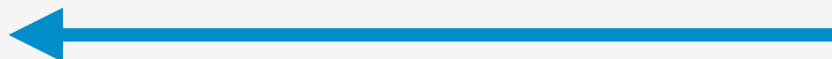
**Adversary**



$pk_{r_1}, nonce_1$



$pre_1$



**Challenger**



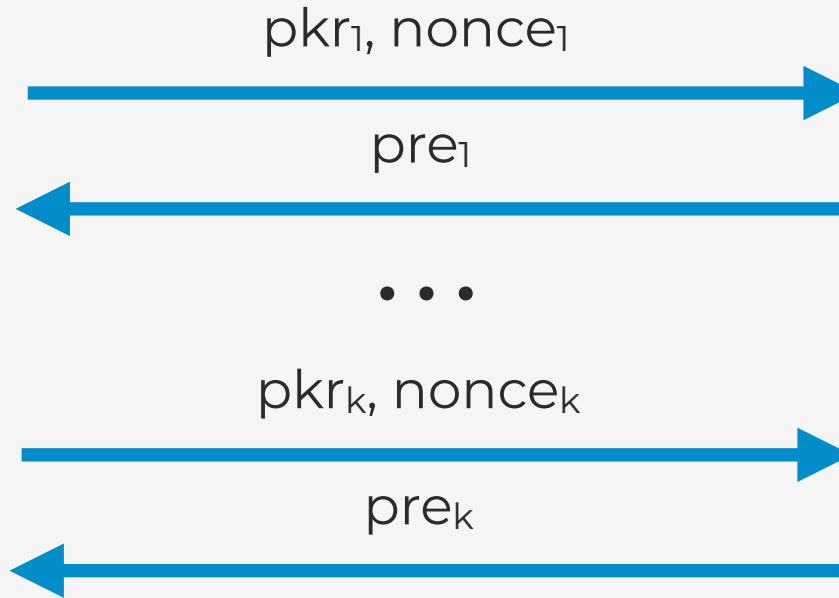


# Unforgeability for NIBS

Adversary



Challenger





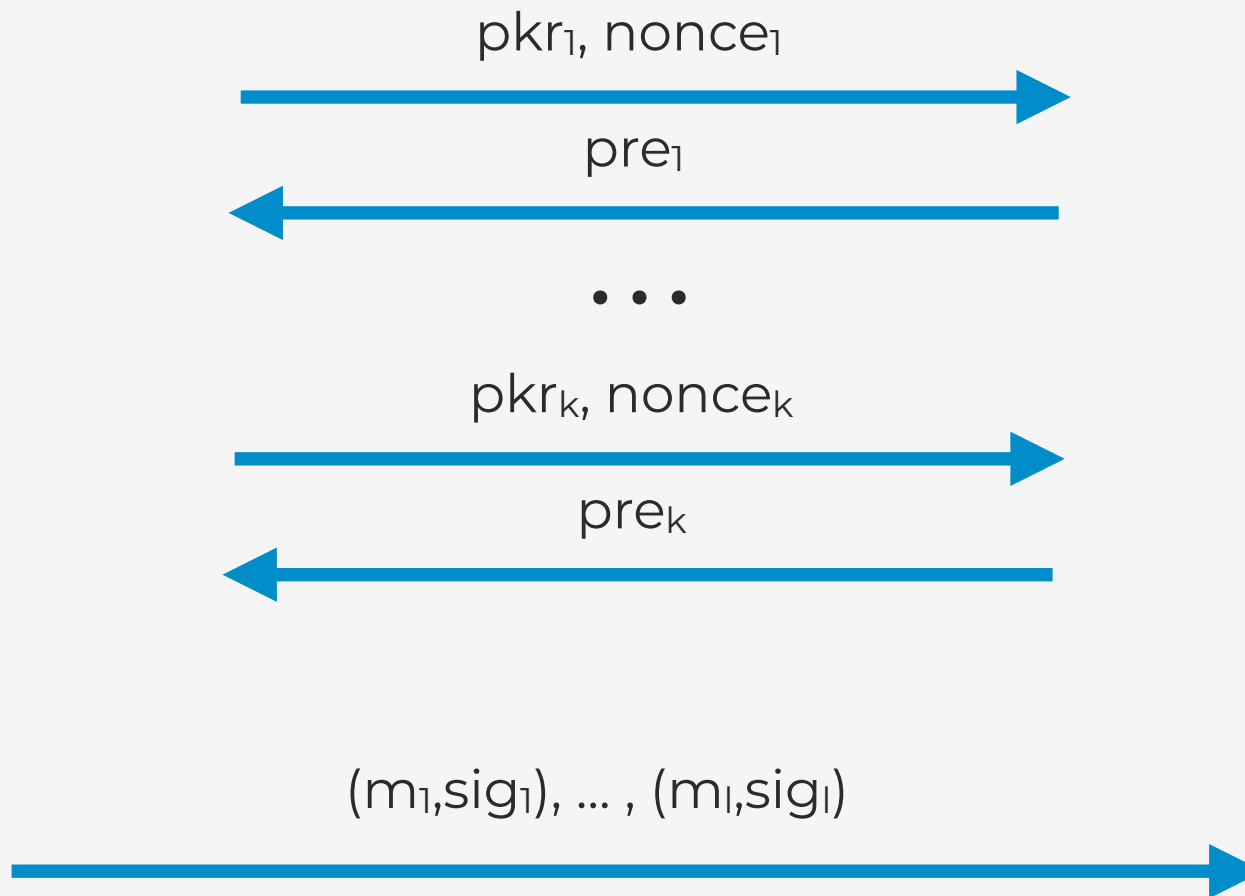


# Unforgeability for NIBS

Adversary



Challenger





# Unforgeability for NIBS

Adversary



Challenger



$pk_{r_1}, nonce_1$

$pre_1$

...

$pk_{r_k}, nonce_k$

$pre_k$

$(m_1, sig_1), \dots, (m_l, sig_l)$

- 1) valid signatures
- 2) distinct messages
- 3) queries  $k < l$



# Blindness for NIBS

## Recipient Blindness

Signatures obtained by different recipient are unlinkable.

Preserves the privacy across recipients.

Recipient Blindness

## Nonce Blindness

Signatures for the same recipient are unlinkable.

Allows to issue multiple presignatures without breaking blindness.

Nonce Blindness



# Recipient Blindness

**Challenger**



**Adversary**





# Recipient Blindness

Challenger



$pk_{r_0}, pk_{r_1}$



Adversary





# Recipient Blindness

Challenger



$pk_{r_0}, pk_{r_1}$



Adversary



$pre_0, nonce_0, pre_1, nonce_1, pk$





# Recipient Blindness

Challenger



$pk_{r_0}, pk_{r_1}$



Adversary



$pre_0, nonce_0, pre_1, nonce_1, pk$



$(m_0, sig_0) \leftarrow \text{Obtain}(sk_{r_0}, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(sk_{r_1}, pk, pre_1)$



# Recipient Blindness

Challenger



Adversary



$pk_{r_0}, pk_{r_1}$



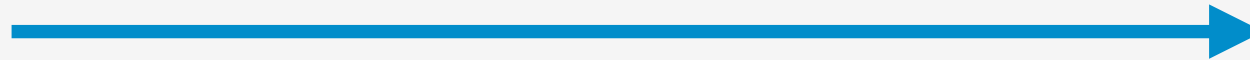
$pre_0, nonce_0, pre_1, nonce_1, pk$



$(m_0, sig_0) \leftarrow \text{Obtain}(skr_0, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(skr_1, pk, pre_1)$

$(m_b, sig_b), (m_{1-b}, sig_{1-b})$







# Recipient Blindness

Challenger



Adversary



$pk_{r_0}, pk_{r_1}$

$pre_0, nonce_0, pre_1, nonce_1, pk$

$(m_0, sig_0) \leftarrow \text{Obtain}(sk_{r_0}, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(sk_{r_1}, pk, pre_1)$

$(m_b, sig_b), (m_{1-b}, sig_{1-b})$

$b'$



# Recipient Blindness

Challenger



Adversary



$pk_{r_0}, pk_{r_1}$

$pre_0, nonce_0, pre_1, nonce_1, pk$

$(m_0, sig_0) \leftarrow \text{Obtain}(sk_{r_0}, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(sk_{r_1}, pk, pre_1)$

$(m_b, sig_b), (m_{1-b}, sig_{1-b})$

**Adversary wins if  
 $b' = b$**

$b'$



# Nonce Blindness

Challenger



Adversary





# Nonce Blindness

Challenger



pk<sub>r</sub>



Adversary





# Nonce Blindness

Challenger



$pk_r$



Adversary



$pre_0, nonce_0, pre_1, nonce_1, pk$





# Nonce Blindness

Challenger



Adversary



pk<sub>r</sub>

pre<sub>0</sub>, nonce<sub>0</sub>, pre<sub>1</sub>, nonce<sub>1</sub>, pk

$(m_0, sig_0) \leftarrow \text{Obtain}(skr, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(skr, pk, pre_1)$



# Nonce Blindness

Challenger



Adversary



$pk_r$

$pre_0, nonce_0, pre_1, nonce_1, pk$

$(m_0, sig_0) \leftarrow \text{Obtain}(skr, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(skr, pk, pre_1)$

$(m_b, sig_b), (m_{1-b}, sig_{1-b})$



# Nonce Blindness

Challenger



Adversary



pk<sub>r</sub>

pre<sub>0</sub>, nonce<sub>0</sub>, pre<sub>1</sub>, nonce<sub>1</sub>, pk

$(m_0, \text{sig}_0) \leftarrow \text{Obtain}(\text{skr}, \text{pk}, \text{pre}_0)$

$(m_1, \text{sig}_1) \leftarrow \text{Obtain}(\text{skr}, \text{pk}, \text{pre}_1)$

$(m_b, \text{sig}_b), (m_{1-b}, \text{sig}_{1-b})$

b'





# Nonce Blindness

Challenger



Adversary



$pk_r$

$pre_0, nonce_0, pre_1, nonce_1, pk$

$(m_0, sig_0) \leftarrow \text{Obtain}(skr, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(skr, pk, pre_1)$

$(m_b, sig_b), (m_{1-b}, sig_{1-b})$

**Adversary wins if  
 $b' = b$**

$b'$



# Preliminaries

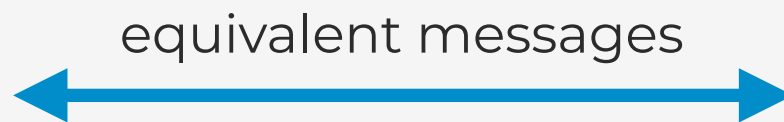
## Signatures on Equivalence Classes



# Preliminaries

## Signatures on Equivalence Classes

$$m = (g^a, g^b)$$



$$m' = (g^{ra}, g^{rb})$$



# Preliminaries

## Signatures on Equivalence Classes





# Preliminaries

## Signatures on Equivalence Classes



**Random  
signature even if  
signer malicious**



# How to efficiently construct NIBS?

$skr^{-1}$

$skr^{-1}$



# How to efficiently construct NIBS?

**Issue(sk, pkr, nonce)**

$\text{pre} := \text{eqsig}(\text{pkr}, H(\text{nonce}))$

$\text{skr}^{-1}$

$\text{skr}^{-1}$



# How to efficiently construct NIBS?

**Issue(sk, pkr, nonce)**

$\text{pre} := \text{eqsig}(\text{pkr}, H(\text{nonce}))$

$\text{skr}^{-1}$

**Obtain(skr, pk, pre, nonce)**

$\text{sig} := \text{adapt}(\text{pre}, \text{skr}^{-1})$

$m := H(\text{nonce})^{\text{skr}^{-1}}$





# How to efficiently construct NIBS?

## Issue(sk, pkr, nonce)

$pre := eqsig(pk_r, H(\text{nonce}))$

## Obtain(skr, pk, pre, nonce)

$sig := adapt(pre, skr^{-1})$

$m := H(\text{nonce})^{skr^{-1}}$

sig is actually  
 $eqsig(g, H(\text{nonce})^{skr^{-1}})$



# How to efficiently construct NIBS?

## Issue(sk, pkr, nonce)

$pre := eqsig(pk_r, H(\text{nonce}))$

## Obtain(skr, pk, pre, nonce)

$sig := adapt(pre, skr^{-1})$

$m := H(\text{nonce})^{skr^{-1}}$

sig is actually  
 $eqsig(g, H(\text{nonce})^{skr^{-1}})$

**pk<sub>r</sub> is a standard  
DH key!**



# Why does it work?

- I. Unforgeability from  
signatures on equivalence signatures
  
- II.  $H(\text{nonce})^{skr^{-1}}$   
is a PRF for the recipient's key
  
- III. Blindness follows from inverse DDH



# Can we Date NIBS? YES!

- I. Signer can add a tag to presignatures that will be preserved
- II. Security notions can be easily adapted to include the tag
- III. Same construction can be used but with tag-based signatures on equivalence classes [HS21]

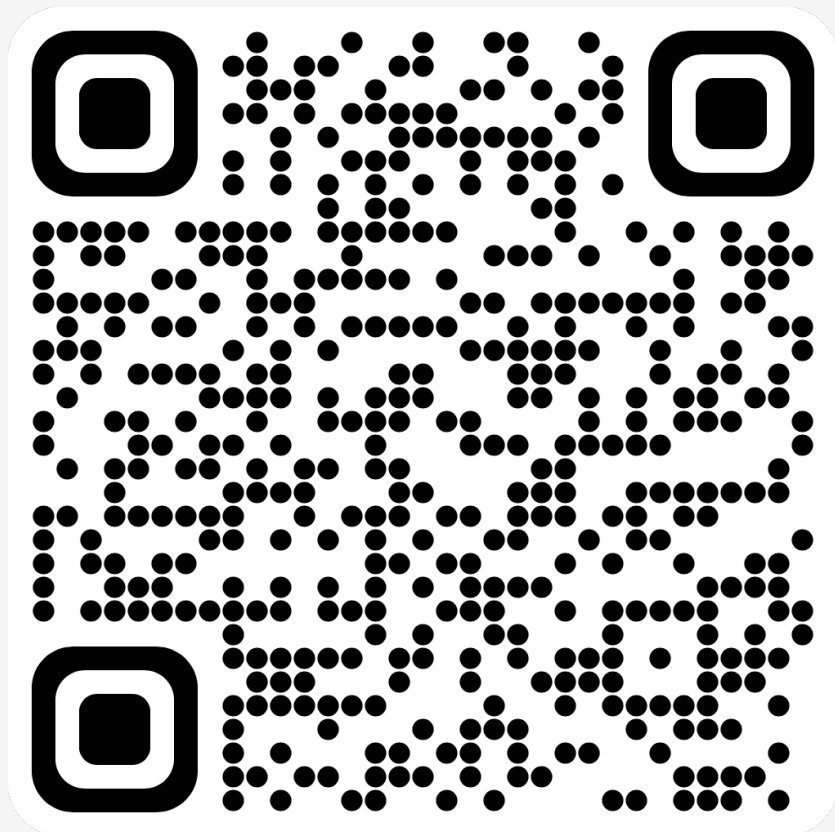


# Summary and Open Problems

- NIBS and TNIBS definitions
  - Efficient constructions that **work with standard PKI keys**
  - Generic construction from VRF and NIWI in ROM
- 
- **Can we construct PQ NIBS/TNIBS?**
  - **Can we construct NIBS/TNIBS without ROM?**

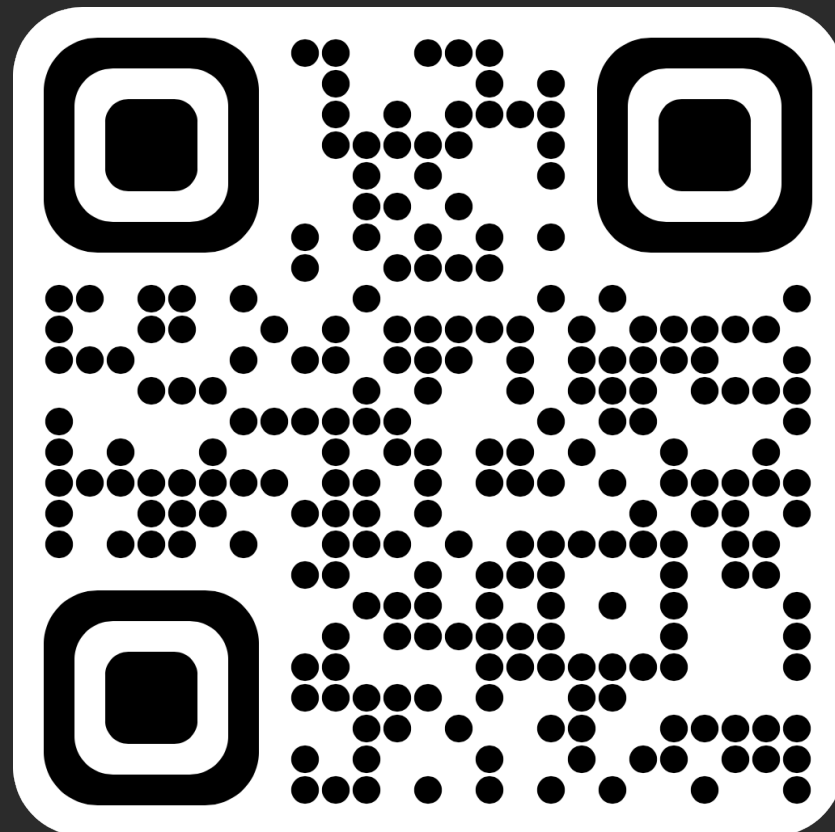


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